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AN APPROXIMATE METHOD FOR CALCULATING THE COLLAPSING PRESSURE OF THIN CYLINDRICAL, CONICAL, AND SPHERICAL SHELLS UNDER UNIFORM EXTERNAL PRESSURE.

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/Note: The following represents a report read at a lecture meeting of the Zōsen Kyōkai /Shipbuilding Association/, held 17 November 1940/.

Author's English-Language Abstract - The author deduces the approximate formulae for calculating the collapsing pressure of conical and spherial shells under uniform external pressure from the exact formulae for cylindrical shells, and shows that the results of collapsing experiments relating to elastic stability of mild-steel shells of the above-mentioned three typical shapes can be represented by a single curve.

Author's Japanese-Language Abstract - The author has contributed over the past ten or so years to experimental research on the collapse, due to external pressure, of thin-wall cylindrical, conical, and spherical shells. In this treatise the author describes a convenient method for expressing compactly the results of these experiments, which he devised during this

period of time. At the end he appends assembled illustrations of all results of experiments on various mild-steel shells, which had been conducted according to this method mentioned above.

(I) <u>Simple Formula for the Stress Intensity and Collapsing Pressure of a Thin-Wall Cylinder.</u>

The author has already related several times the results of his researches on this subject at lecture meetings of this Association and on other occasions. In his earliest report (1) he expressed, as below, the theoretical value of p_k, the pressure at the time when a thin-wall cylinder with strong lid attached at both ends is subjected from all sides to uniform external pressure and consequently collapses because of elastic instability:

$$p_k = \frac{E\frac{s}{D}}{n^2 - 1 + \frac{1}{2}\alpha^2}\left[\frac{2\alpha^4}{(n^2 + \alpha^2)^2} + \frac{2}{3}\cdot\frac{1}{1 - \sigma^2}\left(\frac{s}{D}\right)^2\cdot\left\{\begin{matrix}\text{see}\\ \cdots\cdots\\ \text{below}\end{matrix}\right\}\right]$$

$$\left\{\cdots\right\} \equiv \left\{(n^2 + \alpha^2)^2 - \frac{n^4(2n^2 - 1)}{(n^2 + \alpha^2)^2}\right\}, \qquad (1)$$

where p_k = collapsing pressure

S = thickness of the sylinder shell

D = average diameter of the cylinder

ℓ = unsupported length [occurs in the ratio $\alpha = \frac{\pi D}{2\ell}$]

E = the material's modulus of elasticity

σ = Poisson's ratio

n = the number of furrows (corrugation lines)

$\alpha = \frac{\pi D}{2\ell}$

Now α^2 can be disregarded in comparison with unity since ℓ is fairly large (obviously we can also discard α^2 in comparison with n^2 since $n \geqslant 2$); therefore formula (1) can assume the following form:

$$p_k = \frac{E\frac{s}{D}}{n^2-1}\left[\frac{2\,\alpha^4}{n^4} + \frac{2}{3}\cdot\frac{1}{(1-\sigma^2)}\cdot\left(\frac{s}{D}\right)^2\cdot(n^2-1)^2\right]. \qquad (2)$$

This simplifies the collapsing-pressure formula in the case of cylindrical shells of the above-mentioned measurements ($\alpha^2 \ll 1$; $\alpha^2 \ll n^2$).

If we know the ratio $\frac{s}{D}$ from the measurements of the cylindrical shell, given above, and if we describe according to formula (2) the relation between unsupported length ℓ and collapsing pressure p_k for various corrugation numbers (that is, $n = 2, 3, 4, \ldots$), then we can draw a single curve for each value of n, thus getting a family of curves. Thus in the case of a cylinder whose actual value ℓ is given the value at which collapse occurs corresponds to the minimum value among the various p_k; clearly the corrugation number n also must correspond to this value.

Consequently if the envelop of this family of curves is drawn beforehand, then the ~~intersections~~ differences (of ordinate values) between the effective parts of each curve and the envelop are few; therefore we can by this seek to keep from restricting the collapsing pressure p_k on the corrugation number n.

Here, in finding the envelop of the p_k curves, we determine n from the expression $\partial p_k / \partial n = 0$ and insert it into the main formula [3].

That is, we get approximately

$$n^2 = \alpha \cdot \sqrt[4]{\frac{9(1-\sigma^2)}{(\frac{s}{D})}}\ , \text{ or } n = \sqrt[4]{\frac{\frac{3}{4}\pi^2 \cdot (1-\sigma^2)^{\frac{1}{2}}}{(\ell/D)^2 (s/D)}} \tag{3}$$

Inserting this value of n into the main formula we obtain the following formula for the envelope:

$$p_k = \frac{4\pi}{3\sqrt{3}} \cdot \frac{1-\frac{3}{4}\sigma^2}{(1-\sigma^2)^{3/4}} \cdot E\left(\frac{D}{\ell}\right)\left(\frac{s}{D}\right)^{2.5} \tag{4}$$

Looking at the form of these formulas we see that the main formula (2) is a binomial containing $(\frac{s}{D})$ and $(\frac{s}{D})^3$, whereas formula (4) is of a form containing only the single term $(s/D)^{2.5}$; therefore, according to the main formula (2), if we take (D/ℓ) along the axis of abscissae and draw the p_k curves we can obtain a single curve for each value of (s/D), with p_k finally appearing as a family of curves. But, according to formula (4), if we take $(D/\ell)\cdot(s/D)^{2.5}$ along the axis of abscissae and draw the p_k curves, then no matter what the value of (s/D) only one curve will result.

Now if we take $\sigma = 0.3$ in the case of mild steel, p_k becomes as follows:

$$p_k = 2.4E\,(\frac{D}{\ell})\cdot(\frac{s}{D})^{2.5}\ . \tag{5}$$

If we also use formula (3), then the number n of corrugations that arise in surrounding walls becomes the integer closest to the value determined by the following formula:

$$n = \sqrt[4]{\frac{7.06}{(\ell/D)^2 (s/D)}}\ . \tag{6}$$

Thus the approximate value f_k of the hoop stress intensity arising in cylindrical wall at this time is $f = pD/2S$; therefore from formula (5) we get f_k as follows:

$$f_k = 1.2E\left(\frac{D}{\ell}\right)\cdot\left(\frac{s}{D}\right)^{1.5} \tag{7}$$

(II) Approximate Formula For the Collapsing Pressure Intensity of Thin-Wall Conical Shells [2].

It is not at all possible to solve purely theoretically, as in the case of the cylinder, the problem concerning the elastic stability of a thin-wall cone subjected to external pressure. (A. Flugger. Stabilität dünner Kegelschalen. Ingenieur-Archiv, 1937).

However, results in the conical problem are complicated and are remarkably lacking in practicality.

Accordingly the author, as he related at the December 1931 Congress of the Applied Mechanics League [Ōyō Rikigaku Rengō], solved this problem approximately, by borrowing the formula for the collapsing pressure of a cylindrical shell and by assuming a conical shell which has the same stability as the cylinder.

If we first take one generatrix of the conical surface and consider it as a single rectilinear beam, fixing it firmly at the point corresponding to the apex and merely supporting the end corresponding to the bottom, and next assume that it is loaded with a hoop stress varying according to length (see Figure 1), then the maximum bend will occur at a place about 0.6ℓ from the apex (the length of the generatrix is designated by ℓ). We assume

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for the time being that this is the point of maximum hoop compression along the conical generatrix. If next we construct the normal plane to the conical surface at a point A which is at a distance 0.6ℓ from the apex on the generatrix, then this normal plane and the conical surface form a conic curve (section). Then erecting a normal to the conical surface at this point A, we assume that it interects the axis of the cone at B. In this way we obtain AB, the radius of curvature of the conical curve at point A. Therefore, as an approximate method for solving the problem of collapse in a perfect conical shell, we can consider the cone's collapsing pressure to be the limiting pressure p_k relative to the collapse of a thin-wall cylinder whose length is ℓ and which also possesses a radius $r = AB$ equal to the radius of curvature.

That is, we have:

$$p_k = 2.4E\left(\frac{d}{\ell}\right)\left(\frac{s}{d}\right)^{2.5} \qquad (8)$$

where

$d = 2r = 0.6\ D/\cos\theta$ (D = diameter of the bottom).

The number of wrinkles (corrugations) n_1 generated in this case on the conical surface equals the integer which is closest to the value expressed by the following formula:

$$n_1 = \frac{2\pi r_1}{\left(\frac{2\pi r}{n}\right)} = \frac{2\pi r\cdot\cos\theta}{\left(\frac{2\pi r}{n}\right)} = n\cdot\cos\theta \qquad (9)$$

Also, the hoop stress intensity which occurs at this place of maximum compression is given by:

$$f_k = 1.2E \left(\frac{d}{l}\right) \left(\frac{s}{d}\right)^{1.5} \tag{10}$$

(III) Approximate Formula for the Collapsing Stress Intensity of Thin-Wall Spherical Shells (3).

It seems that the complete solution of the collapse of a sphere still cannot be obtained theoretically. At least there are no solutions that can be applied practically. Therefore the author endeavored to derive an approximate formula for the collapsing pressure of a spherical shell by means of new ideas.

Since the sphere possesses perfect symmetry relative to the center, the shape of the sphere at the instant of collapse must also be symmetrical relative to the center. Therefore the proper procedure to follow is to assume that the shape at the very instant of the spherical shell's collapse is a regular polyhedron. In short, the hollow depressions at the moment that the sphere collapses are created in the positions that correspond to the facies (hedra) of the regular polyhedron. Now let us consider a regular polyhedron such that each edge touches the sphere. (This polyhedron can be considered to possess that shape which is closest to a sphere located between the inner contiguous (inscribed) sphere and the outer contiguous (circumscribed) sphere.) We can approximate it by means of conical shapes which contain as the generatices the edges which collect at an apex of this polyhedron. Now, as is well known, what we call regular polyhedra can only be the five regular geometrical solids possessing 4, 6, 8, 12, 20 facies (tetra-, hexa-, octa-, dodeca-, icosa- hedra). The problem is which of these five should be taken to approximate a sphere at collapse. In considering this problem the author attempted to select two reasonable possibilities. (See Appendix II). At any rate, the figure most suitable for this case among the above-mentioned regular polyhedra was finally decided to be the regular icosahedron (20 facies). According to the information (see Figure 2) related in detail in reference (3) of Appendix I, we have:

$$\ell = 0.31\ D,$$

if d = 0.6 D is the diameter (where D is the average diameter of the spherical shell). Therefore, since $d/\ell = 1.94$, we may set s/0.6 D in place of s/D and $d/\ell = 1.94$ in equation (5) giving the collapsing pressure of a cylindrical shell. That is, we have:

$$p_k = 2.4E \times 1.94\ \left(\frac{s}{0.6D}\right)^{2.5} = 2.4E\ \left(\frac{4.17}{0.6}\right)\cdot\left(\frac{s}{D}\right)^{2.5} = 16.70E\ \left(\frac{s}{D}\right)^{2.5}; \qquad (11)$$

$$f_k = \frac{p_k \times 0.6\ D}{2s} = \frac{p_k\ D^*}{3.34s} = \frac{16.70}{3.34} \times E\ \left(\frac{s}{D}\right)^{1.5} = 5.0\ E\ \left(\frac{s}{D}\right)^{1.5}. \qquad (12)$$

(*Note: As is well known, the hoop stress intensity of a spherical shell is theoretically f = pD/4s. The difference between this and the preceeding value merely results from our employing the approximate derivation discussed above.)

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(*Note: As is well known, the hoop stress intensity of a spherical shell is theoretically $f = pD/4s$. The difference between this and the preceeding value merely results from our employing the approximate derivation discussed above.)

(IV) Convenient Method for Expressing Compactly the Results of Experiments on the Collapse Intensity of Various Shells.

We finally obtain the following compact table expressing the approximate formulas for the hoop stress intensity and collapsing pressure of the cylindrical, conical, and spherical shell:

Shell Type:	Approximate formula for Collapsing Pressure:	Approximate Formula for Hoop Stress Intensity:	Theoretical Formula for Hoop Stress
Cylinder	$p_k = 2.4\ E\left(\frac{D}{\ell}\right)\left(\frac{s}{D}\right)^{2.5}$	$f_k = 1.2\ E\left(\frac{D}{\ell}\right)\left(\frac{s}{D}\right)^{1.5}$	$f = \frac{pD}{2s}$
Cone	$p_k = 2.4\ E\left(\frac{d}{\ell}\right)\left(\frac{s}{d}\right)^{2.5}$	$f_k = 1.2\ E\left(\frac{d}{\ell}\right)\left(\frac{s}{D}\right)^{1.5}$	$f = \frac{pd^{*}}{2s}$
Sphere	$p_k = 16.7\ E\left(\frac{s}{D}\right)^{2.5}$	$f_k = 5.0\ E\left(\frac{s}{D}\right)^{1.5}$	$f = \frac{pD}{4s}$

(*Note: The place of maximum compression.)

How may we summarize the results obtained from collapsing tests on the various shell types containing the same material and data?

(Of course if they are not made of the same material, the shapes of the curves expressing the stress-strain relation will not be equal; therefore it would be illogical to compare them.)

The author has tentatively formulated the following procedure: Namely, using logarithmic ruled paper, place on the axis of abscissae the values

$(D/\ell)\cdot(s/D)^{1.5}$ (for the cylindrical shell)

$(d/\ell)\cdot(s/d)^{1.5}$ (for the conical shell)

$4.17\cdot(s/D)^{1.5}$ (for the spherical shell);

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next compute the values of the hoop stress intensity f_k - namely,

$$f_k = (p_k\, D/2s)^{*} \quad \text{(for the cylinder)}$$

$$f_k = (p_k\, d/2s)^{*} \quad \text{(for the cone)}$$

$$f_k = (p_k\, D/4s)^{*} \quad \text{(for the sphere)} \text{ —}$$

from the pressure obtained experimentally and place on the ordinate axis. (*Note: We employ the theoretical formulas instead of the approximate formulas in determining the hoop stress.)

(V) Results of Graphing, According to the Above-Described Procedure, Previous Experimental Values; Conclusions.

The various mild-steel shells investigated by the author total 130, divided as follows:

1. 52 cylinders (none were reinforced):
   diameter 600 mm; thickness 2, 3, 5 mm; length, various.
2. 44 conical shells:
   bottom diameter 600 mm; thickness 3,5, 6 mm; height, various.
3. 34 spherical shells:
   diameter 200, 400, 600, 800 mm; thickness 1, 1.5, 2, 2.5, 3, 4, 5, 6 mm.

These values were placed, as in the appended figures (see Figure 3), in proper correspondence to the values on the axis of abscissae on the above-mentioned logarithmic ruled paper; furthermore, I tentatively included also the results of experiments (13) conducted by the German Navy during World War I. As is evident from the figure, the points are distributed comparatively well in a fixed band. If we allow a unit margin of about 15% above and below the mean curve, we obtain results that are completely applicable in practical plans. Especially interesting, as we note, is the fact that this curve plainly suffers an inflection after one passes beyond the point of the structural material's proportional limit (in the case of mild steel, as here, this limit is 1800~1900 kg/cm^2).

The fact that the experimental mean curve is below the approximate theoretical values is due probably to the boundary conditions of the end faces or to the skill (or lack of it) in the construction and inhomogenieties of the material; therefore, the longer the shell the closer the experimental values approach the theoretical values, and up to the limit of proportionality the causes of this decrease in the experimental values may be ascribed to the so-called "imprecision of construction". Above the proportional limit, however, the material enters the plastic stage and therefore the material's elastic modulus changes to confuse the picture. Consequently, if we can decide the stress-strain relation of the material's plastic stage, then this problem too could be clarified, according to the author's opinion. The author is still endeavoring to conduct investigations in connection with this problem.

Finally, the author wishes to take this opportunity to express his feeling of gratitude to the various personnel of the Navy Technical Laboratory (Kaigun Giken) for their assistance and to the Naval authorities concerned in this work, who permitted him to present this paper at this meeting.

Appendix I. Works of Authors Mentioned in This Report.

(The index numbers here are the same as those used in the text.)

(1). "~~Mold~~ Model Experiments on the Elastic Stability of Closed and Cross-Stiffened Circular Cylinder Under Uniform External Pressure." Paper No. 651, World Engineering Congress, November 1929.

(2). "Experiments on the Elastic Stability of Thin-Wall Cone Under Uniform Normal Pressure on All Sides, and An Approximate Method for Computing Its Collapsing Pressure." Published report heard at the Applied Mechanics League's /Ōyō Rikigaku Rengō Taikai/ November 1931, Congress. Shipbuilding Association /Zōsen Kyōkai/ Miscellaneous Publication No. 125 Annex (published August 1932, pages 151-169).

(3). "Experiments on the Collapse of a Thin Sphere Under External Pressure, and an Approximate Method for Computing its Collapsing Pressure." Published report heard at the Shipbuilding Association's Autumn Lecture November 1936 Meeting. Shipbuilding Association Report No. 59 (published December 1937, pages 179-194).

(3)* Ibidem. See Formula - (7) on page 183.

(4). TOKOGAWA Takesada, and KITO Shijo. "Fundamental Theory of Elastic Stability," pages 225-228. Publication of the Shipbuilding Research Section of the Naval Technical Research Laboratories. (Not for sale).

Appendix II. Reasons for Assuming it Proper to Select the Regular Icosahedron From Among the Five Regular Polyhedra When Approximating to the Conditions Governing the Collapse of Thin-Wall Spherical Shells.

(i) The "Cover relation" - namely, the area of the spherical area which is covered by the cone whose apical angle is 2θ - is given by the expression $M = 2\pi r^2 (1 - \sin\theta) = 2\pi hr$ (see Figure 4).

If the number of apices P is taken as n, then it is covered by all the cones.

The area of the spherical area is M.n. If the ratio of this to the entire sphere's area $4\pi r^2$ is provisionally called the "cover ratio", we have:

$$K = \frac{M.n}{\text{entire sphere's area}} = \frac{2\pi r^2 n(1-\sin\theta)}{4\pi r^2} = \frac{n}{2}(1 - \sin\theta).$$

Now, if we compute K relative to the various polyhedra, we get:

Type:	n		K	Shape of the Various Faces
regular tetrahedron	4	35°16'	0.843	regular triangle
regular hexahedron	8	54°44'	0.744	regular square
regular octahedron	6	45°0'	0.879	regular triangle
regular dodecahedron	12	69°5'	0.660	regular pentagon
regular icosahedron	20	58°17'	0.894	regular triangle

If, on the other hand, n is very large - that is, if we consider the case where we insert into the sphere infinitely many very small cones - this becomes equivalent to the problem of arranging approximately round plates on a flat area.

In Figure 5 the parts with the oblique lines - that is, the parts remaining after the courtyard has been eliminated - are as follows:

(a) $K = \frac{\left(\pi r^2 \cdot \frac{60}{360}\right) \cdot 3}{\frac{1}{2}(2r) \cdot 2r \cdot \cos 30^\circ} = 0.906$ in the case of a regular triangle consisting of circular arcs.

(b) $K = \frac{\left(\pi r^2 \cdot \frac{90}{360}\right) \cdot 4}{2r \cdot 2r} = 0.7854$ in the case of a regular square consisting of circular arcs.

(c) $K = \frac{\left(\pi r^2 \frac{72}{360}\right) \cdot 5}{\frac{5}{4} e^2 \sqrt{\frac{3+\sqrt{5}}{5-\sqrt{5}}}} = 0.6875$ [sic] (e is the length of one side of the regular dodecahedron) in the case of a regular pentagon consisting of circular arcs.

The regular hexahedron and regular dodecahedron, in which the parts with "courtyard" excluded are squares and pentagons consisting of circular arcs, are the least from the standpoint of the "cover ratio"; therefore, we have to consider only the problem of the triangle consisting of circular arcs. In every case K must become larger as $n \to \infty$, but the one closest to extreme limit value 0.906 is the value 0.894 belonging to the regular icosahedron.

(ii) If we take the edge of each regular polyhedron as the generatrix and compute tentatively for each regular polyhedron the ratio H/D (= height/bottom diameter) of cones contained, we get:

Polyhedron:	K	H/D
regular tetrahedron	0.843	0.705
regular hexahedron	0.744	0.353
regular octahedron	0.879	0.498
regular dodecahedron	0.660	0.190
regular icosahedron	0.894	0.315

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When we actually examine the appearances and forms assumed during collapse of conical shells, we find that the ratio H/D = 0.367 is located on the inflection point, and the conditions governing collapse corresponding to ratios H/D below this vary from that of (a) in Figure 6 to that (b). When we experimentally study the pattern of collapse in spherical shells, we find that it assumes the appearance shown in (b) of Figure 6.

The ratios of the above-mentioned regular hexahedron and dodecahedron are less than 0.367, but from the point of view of the "cover ratio" there is no problem. The H/D = 0.315 ratio of the regular icosahedron remains as the most suitable value.

According to the above-discussed reasons we must choose the regular icosahedron, and we consider a hypothetical conical body to exist at each apex of this regular icosahedron.

Discussion

Chairman (KUWAHARA Chuji): If there are any questions or opinions relative to this report, please speak. Are there no questions? Well then I should like to ask a little question myself. Did you carry out the limit process in the plate thickness and diameter?

TOKUGAWA Takesada: It was about 0.003, as usual.

Chairman (KUWAHARA Chuji): Any more questions? Since it seems that no one desires to speak, I should like to say a word or two. Dr Tokugawa is a great authority on this problem, and has published numerous reports. Our country too is using the results of researches in its design of submarines, and already very great advantages have been obtained and applied. Now, bring into together the results of long years of research, Mr Tokugawa has constructed simple formulas for computations in connection with cones, cylinders, and spheres, which I think facilitates the work of the actual designer. I wish to express my gratitude that his useful investigations were able to be reported. I am sure that every one will wish to show their appreciation by applauding. (Applause.)

[Figures 1 to 6 follow]

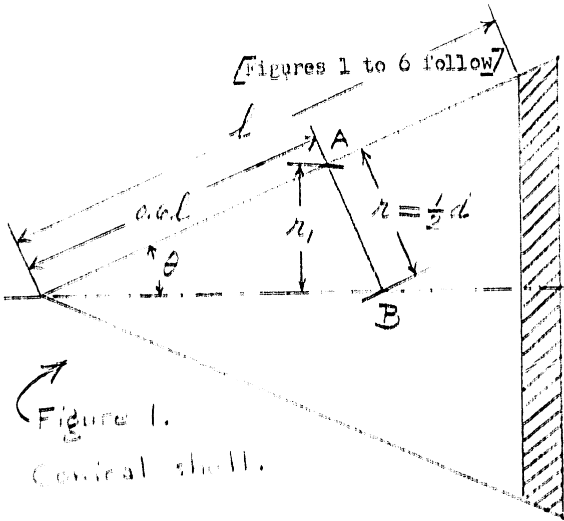

Figure 1.
Conical shell.

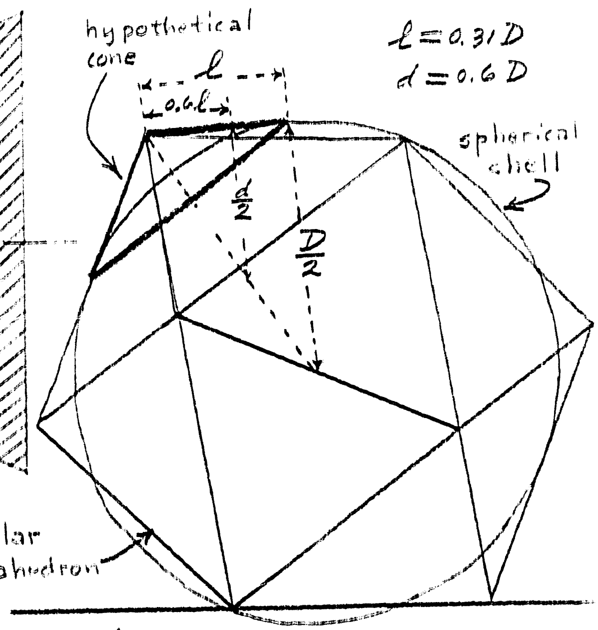

Figure 2.
Spherical shell.

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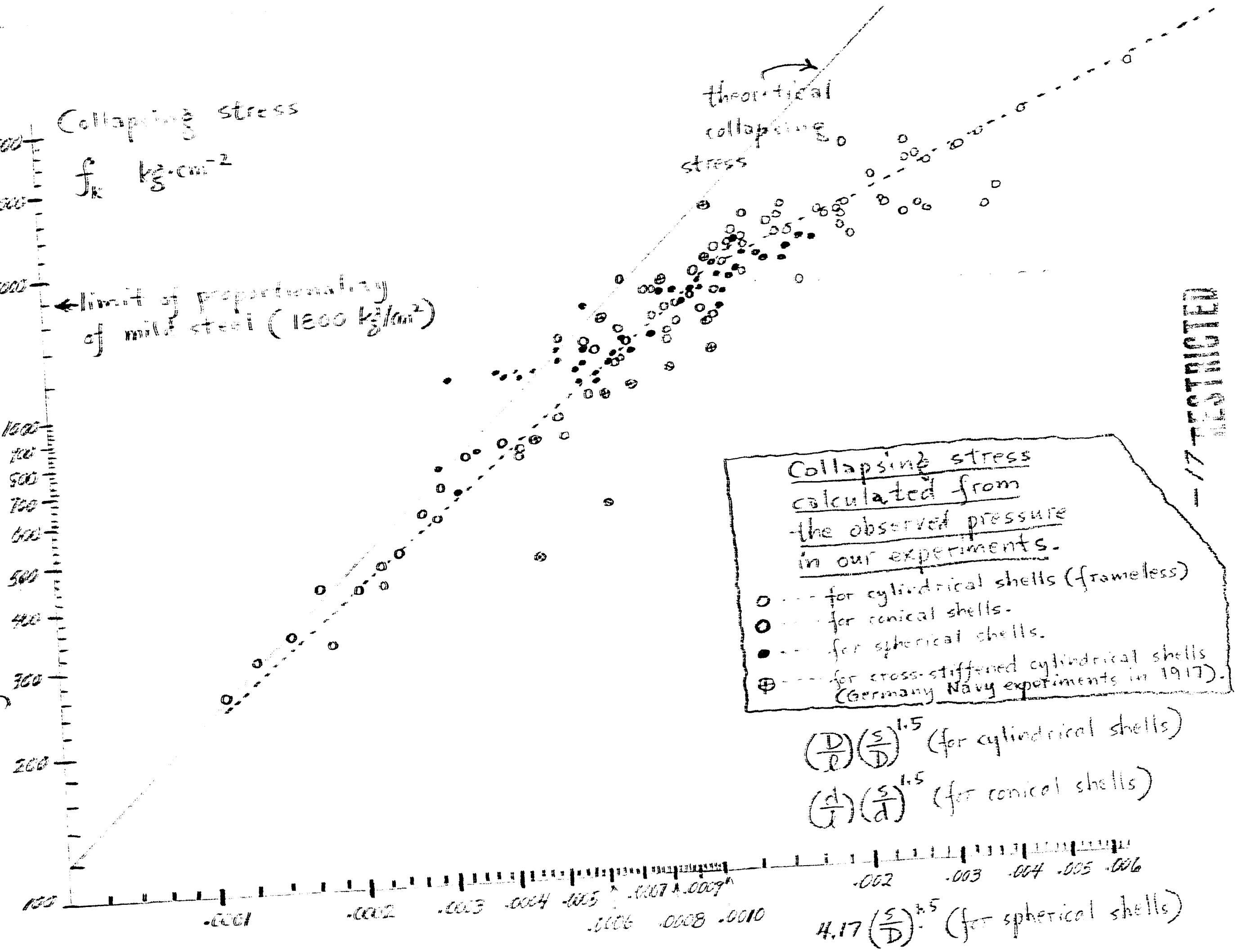

Figure 3. Results of experiments on the collapsing stress of mild-steel thin-wall cylindrical, conical, and spherical shells.

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Figure 4.

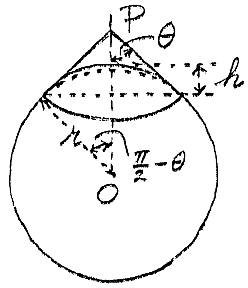
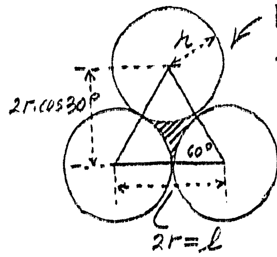


Figure 5 (a)
Regular triangular face



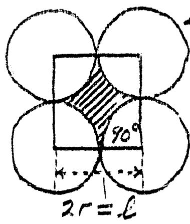
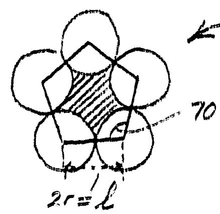


Figure 5 (b)
Regular square

Figure 5 (c)



Regular pentagonal face

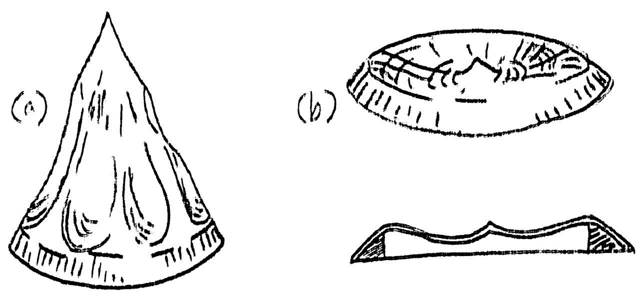


Figure 6. Appearances of Two Types of Collapse in Conical Shells

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徳川武定 TOKUGAWA Takesada
鬼頭史城 KITŌ Shijō
桑原重治 KUWAHARA Chūji

造船協會 Zōsen Kyōkai [Shipbuilding Association]

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